

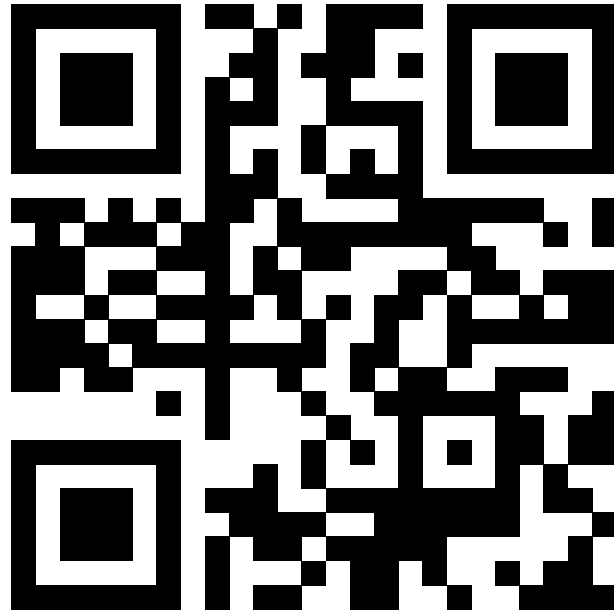
RÉSOLUTION DE PROBLÈMES

COMBINATOIRES

SAT & CDCL

Loïc & Loïc

Prise de notes collaboratives



minifi.ca/rpcl

Menu du jour

- Rappels
- CDCL
- Arbres de recherches
- Comparaisons

Rappels

SAT : Problème de SATisfiabilité Booléenne.

CNF : Conjunctive Normal Form (\wedge de \vee).

DNF : Disjunction Normal Form (\vee de \wedge).

$$\neg \text{CNF} = \text{DNF} \quad \wedge \quad \neg \text{DNF} = \text{CNF}$$

Algorithme *Conflict-Driven Clause Learning* (CDCL)

Idée :

$$\text{clause} \rightarrow \perp \iff \top \rightarrow \neg \text{clause}$$

Exemple

$$x_1 + x_4$$

$$x_1 + x_3' + x_8'$$

$$x_1 + x_8 + x_{12}$$


$$x_2 + x_{11}$$

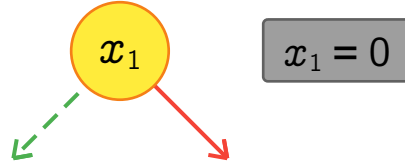
$$x_7' + x_3' + x_9$$

$$x_7' + x_8 + x_9'$$

$$x_7 + x_8 + x_{10}'$$

$$x_7 + x_{10} + x_{12}'$$

$$x_1 = 0$$




Exemple

$$x_1 + x_4$$

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$$x_1 + x_8 + x_{12}$$

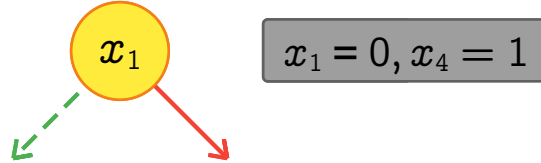
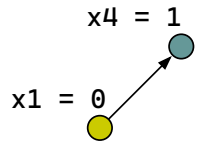
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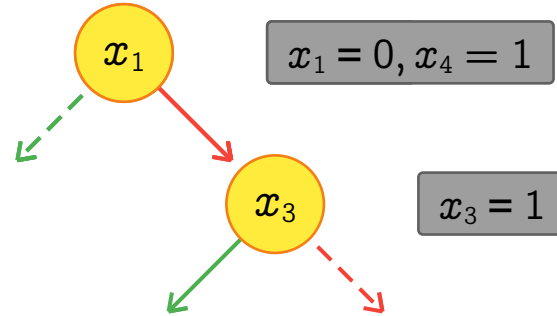
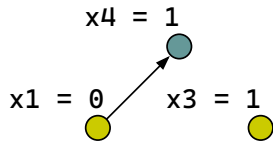
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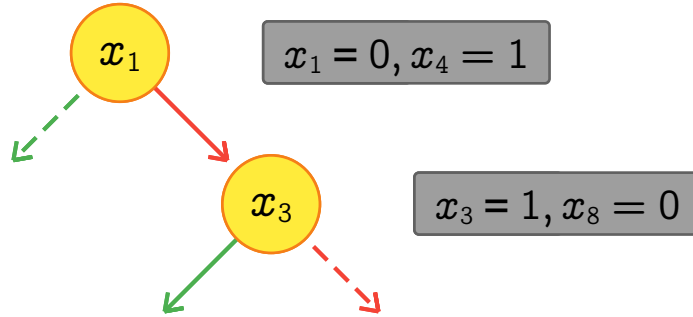
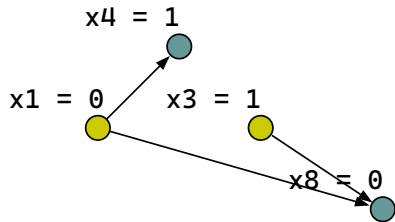
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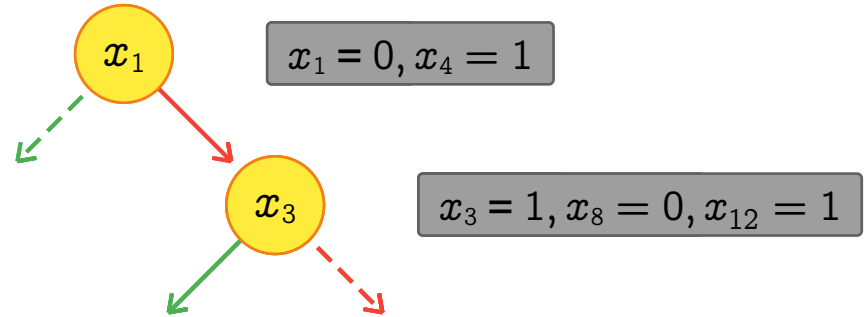
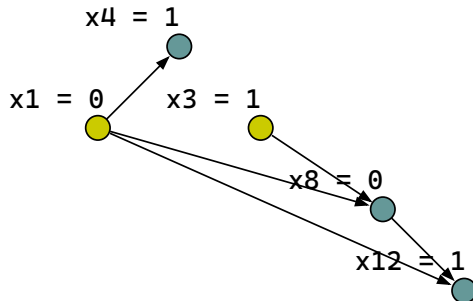
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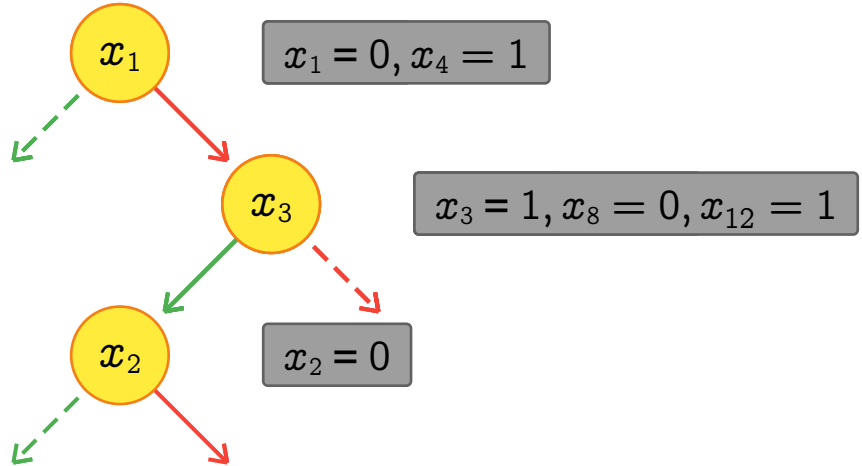
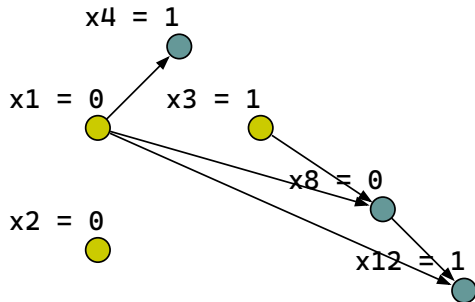
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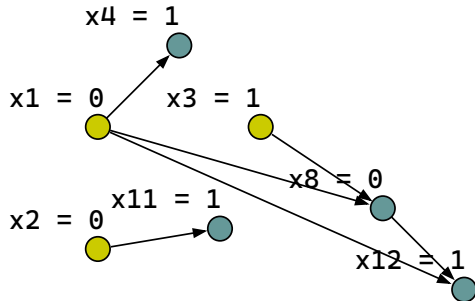
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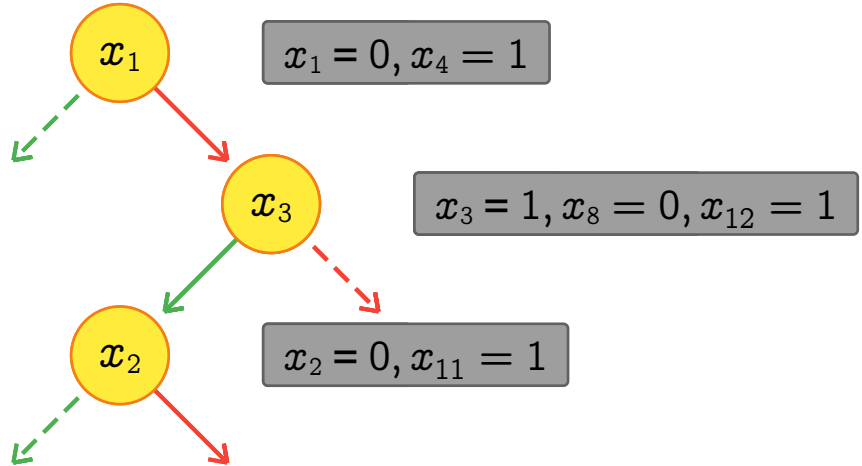
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L.&L.



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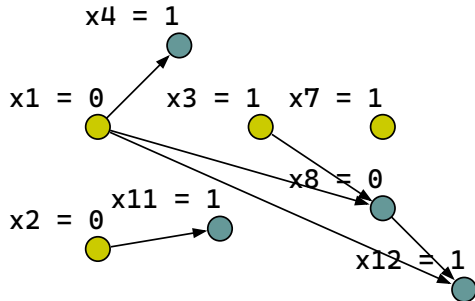
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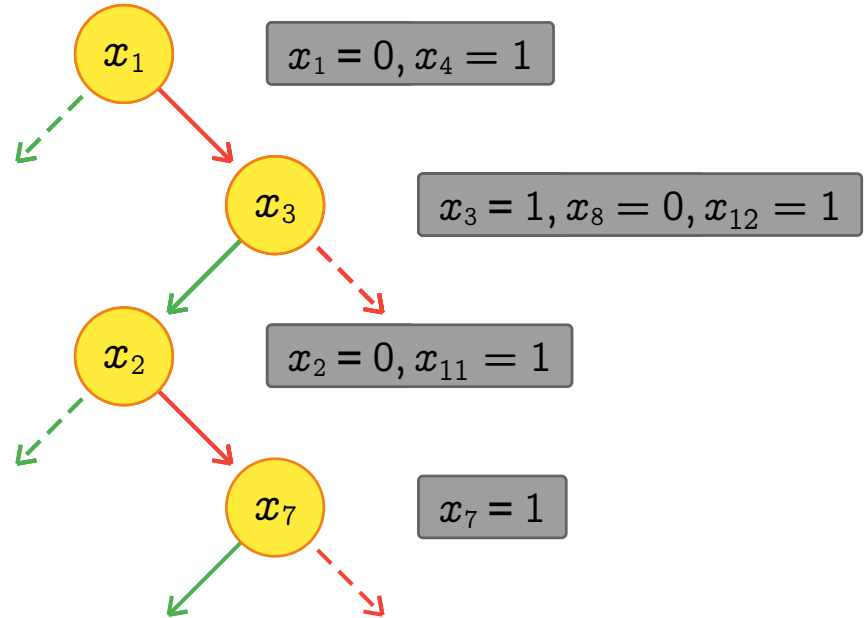
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L.&L.



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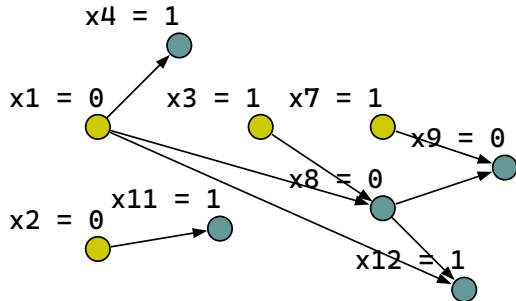
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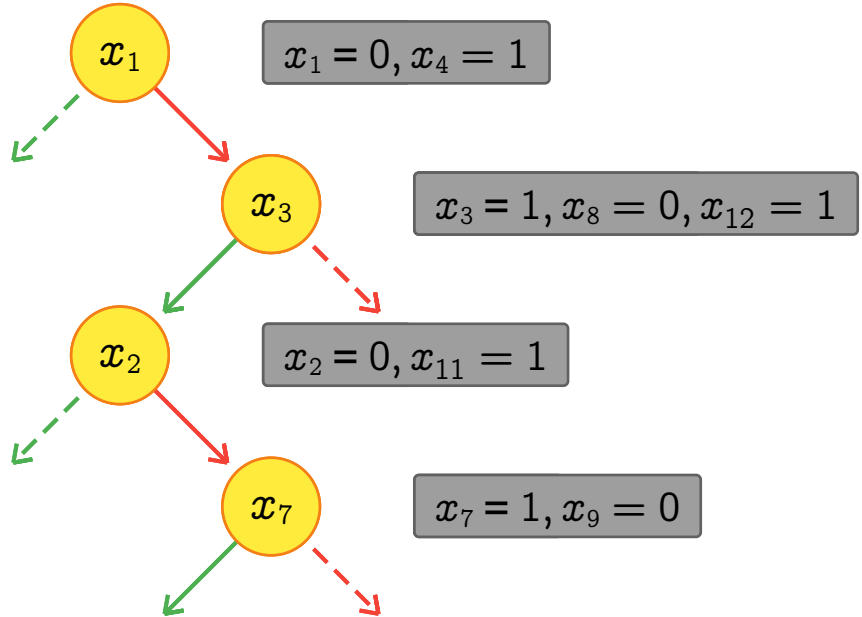
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L.&L.



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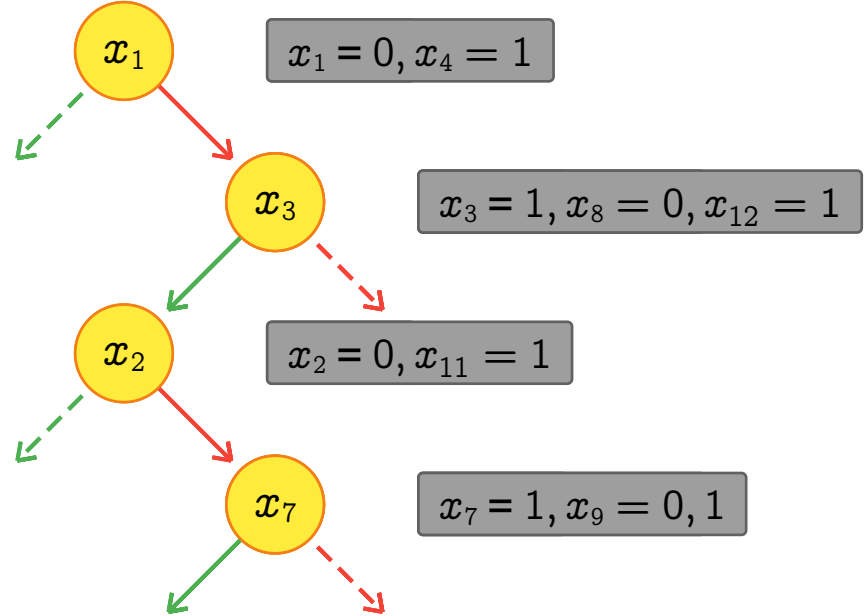
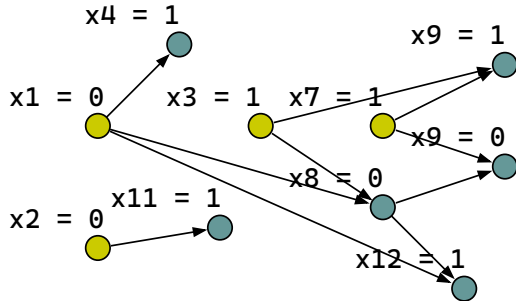
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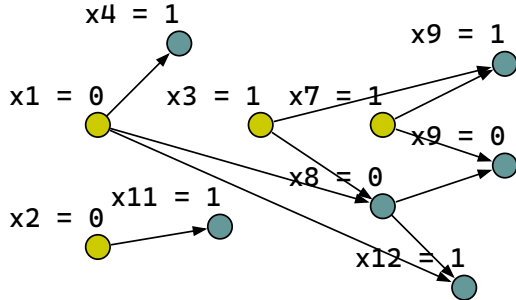
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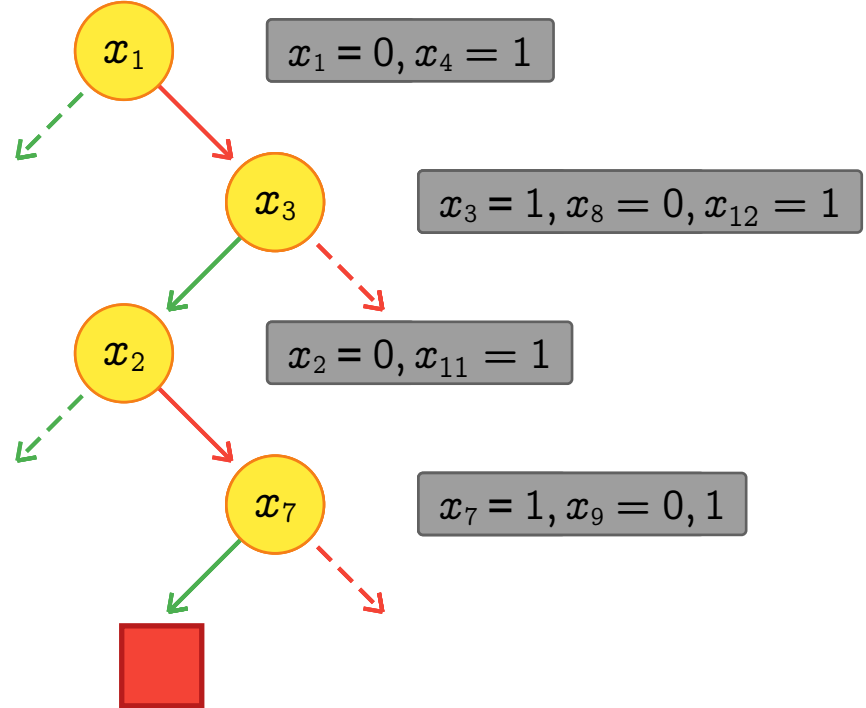
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L.&L.



Example

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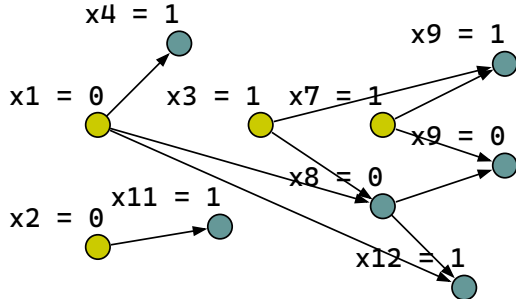
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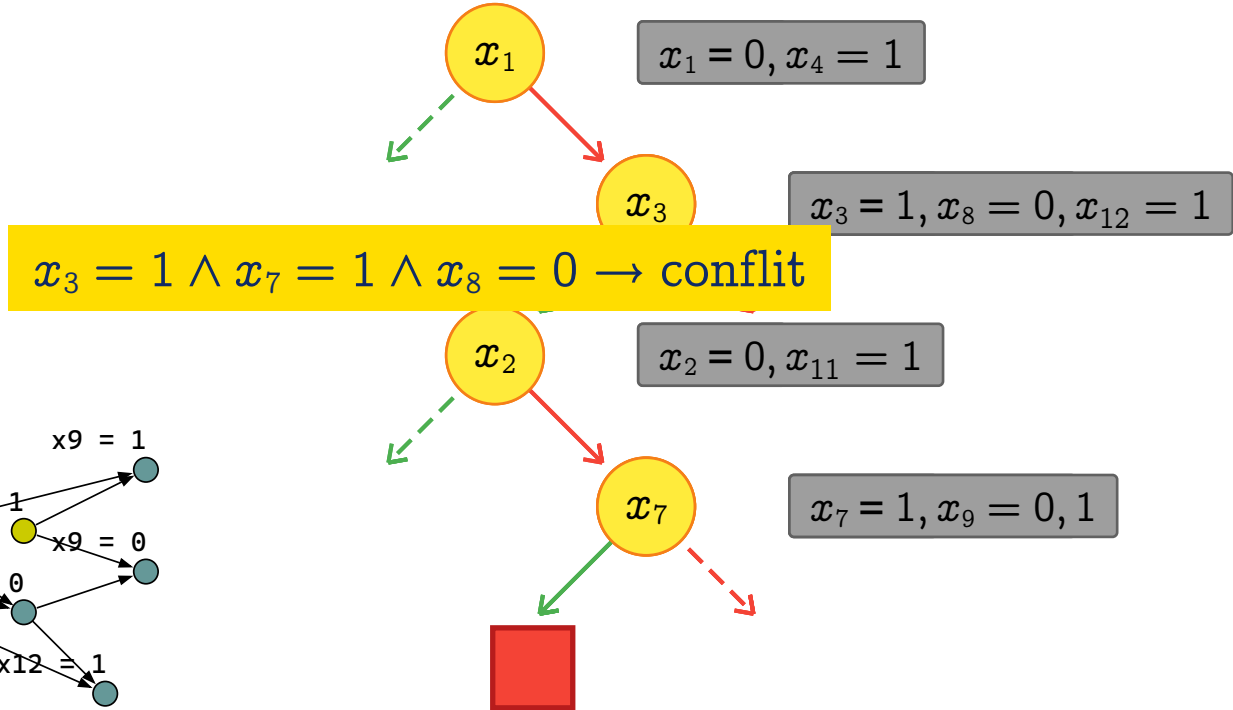
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L.&L.



Exemple

$$x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0 \rightarrow \text{conflit}$$

Exemple

$x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0 \rightarrow \text{conflit}$
pas conflit $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

Exemple

$x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0 \rightarrow \text{conflit}$

pas conflit $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

true $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

Exemple

$x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0 \rightarrow \text{conflit}$

pas conflit $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

true $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

$(x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

Exemple

$x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0 \rightarrow \text{conflit}$

pas conflit $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

true $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

$(x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

$(x_3 = 0 \vee x_7 = 0 \vee x_8 = 1)$

Exemple

$x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0 \rightarrow \text{conflit}$

pas conflit $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

true $\rightarrow (x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

$(x_3 = 1 \wedge x_7 = 1 \wedge x_8 = 0)'$

$(x_3 = 0 \vee x_7 = 0 \vee x_8 = 1)$

$(x_3' + x_7' + x_8)$

Example

$$x_1 + x_4$$

$$x_1 + x_3' + x_8'$$

$$x_1 + x_8 + x_{12}$$

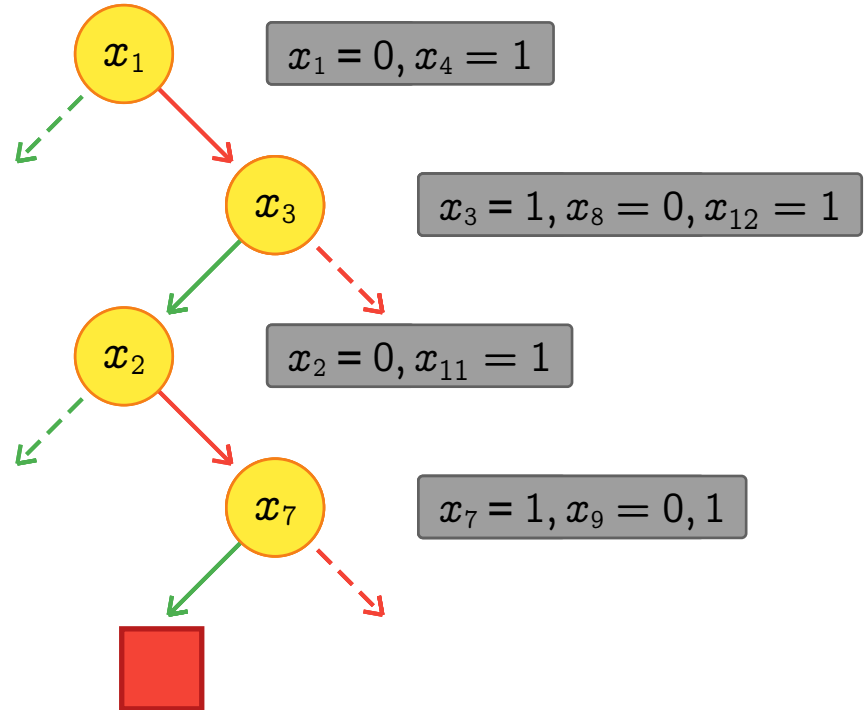
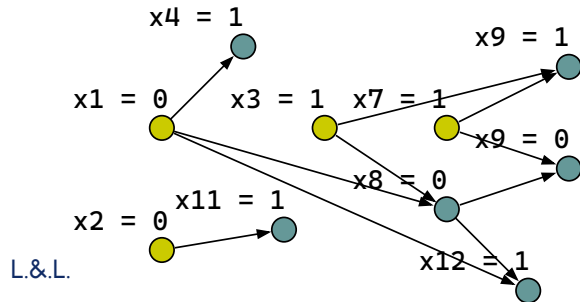
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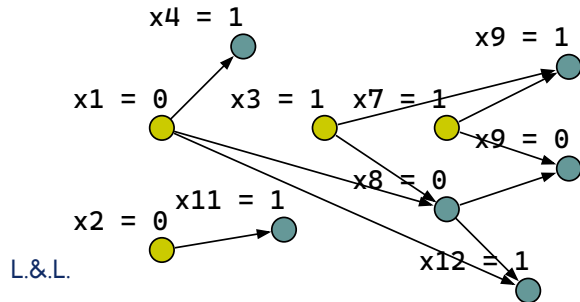
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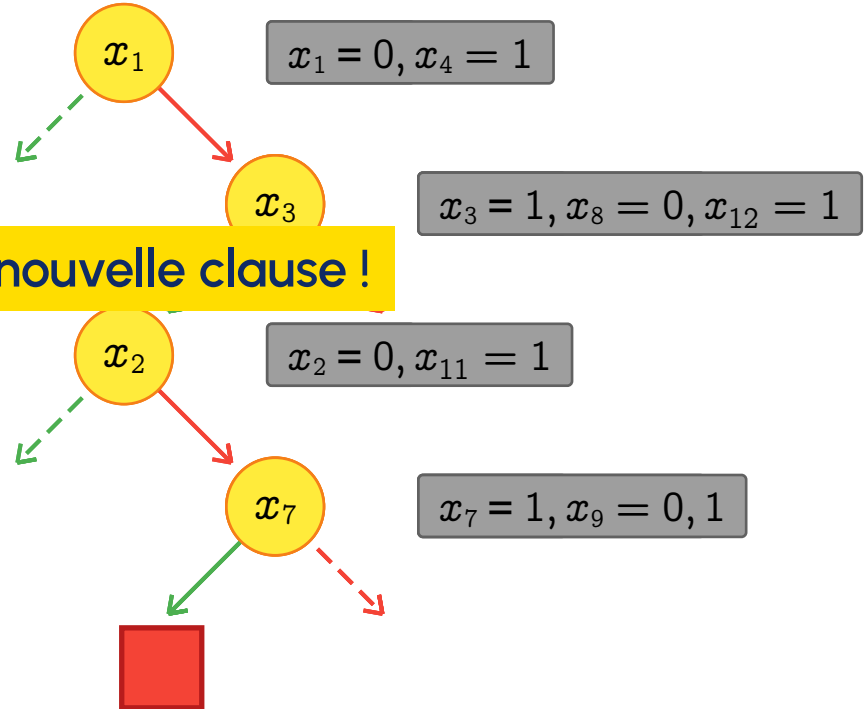
$$x_7' + x_8 + x_9'$$

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Ajout de la nouvelle clause !



Exemple

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$$x_2 + x_{11}$$

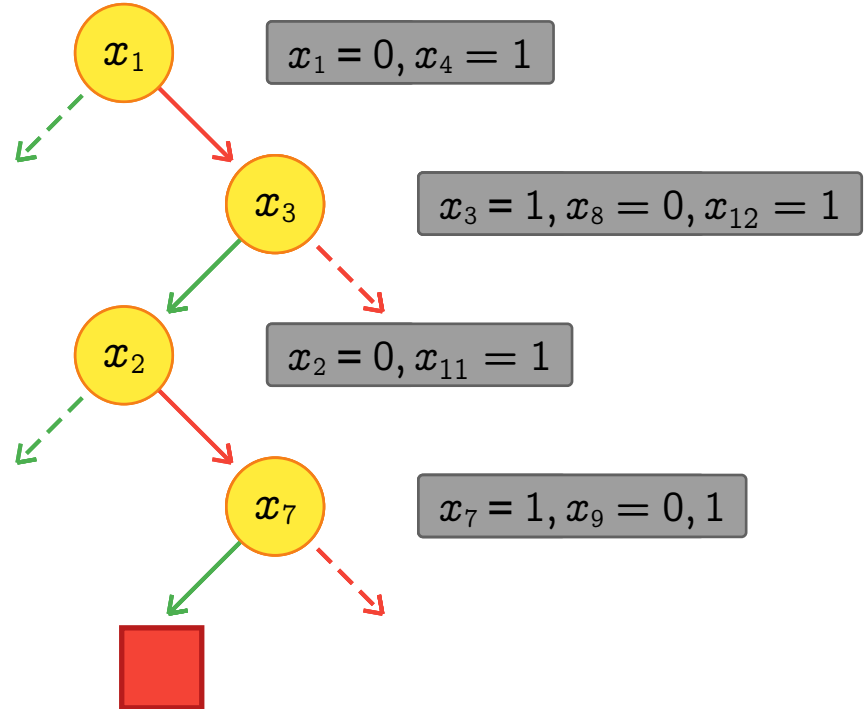
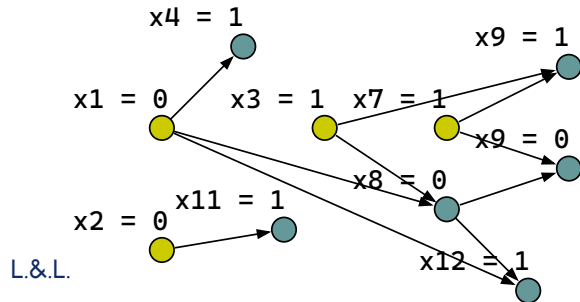
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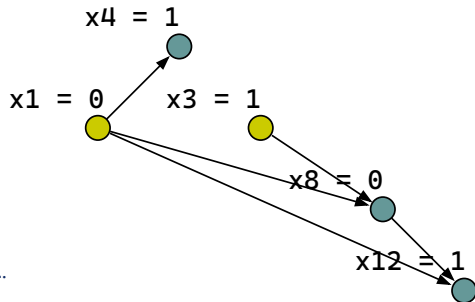
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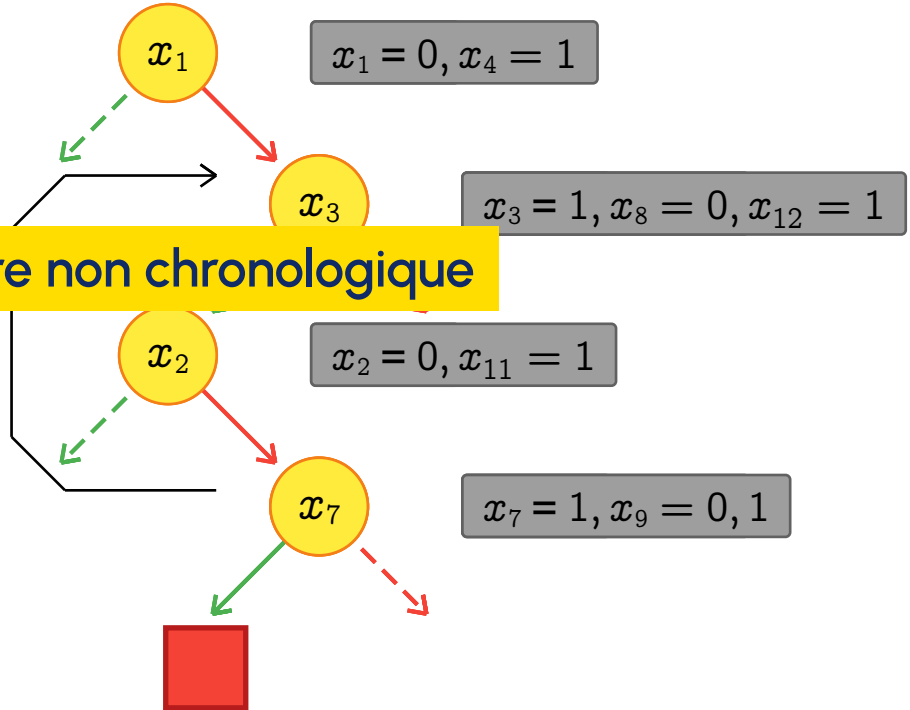
$$x_7 + x_{10} + x_{12}'$$

$$x_3' + x_8 + x_7'$$



L.&L.

Retour arrière non chronologique



Exemple

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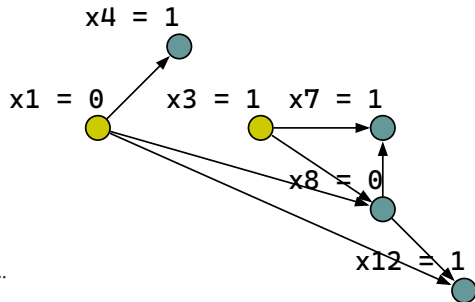
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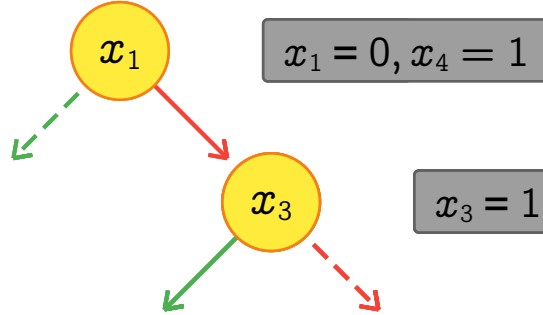
$$x_7 + x_8 + x_{10}'$$

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L.&L.



TODO

1. Finir d'écrire les convertisseurs Sudoku \Rightarrow Dimacs
1. Trouver un moyen de comparer les approches DPLL et CDCL (taille des arbres de recherche) et faire des expérimentations pour voir l'impact des algorithmes sur l'espace de recherche ;
2. Faire des benchmarks du Sudoku sur différents solveurs (CP & SAT).